

HOMework

- Section 5.3 - 49, 50, 53

SECTION 5.3 - LINEAR PROGRAMMING IN TWO DIMENSIONS: A GEOMETRIC APPROACH

The following theorems give us information about the solvability and solution of a linear programming problem:

Theorem 1 (Fundamental Theorem of Linear Programming). *If the optimal value of the objective function in a linear programming problem exists, then that value must occur at one or more of the corner points of the feasible region.*

Theorem 2 (Existence of Optimal Solutions).

- (A) *If the feasible region for a linear programming problem is bounded, then both the maximum value and the minimum value of the objective function always exist.*
- (B) *If the feasible region is unbounded and the coefficients of the objective function are positive, then the minimum value of the objective function exists, but the maximum value does not.*
- (C) *If the feasible region is empty, then both the maximum value and the minimum value of the objective function do not exist.*

Geometric Method for Solving Linear Programming Problems.

Procedure (Geometric Method for Solving a Linear Programming Problem with Two Decision Variables).

- (1) *Graph the feasible region. Then, if an optimal solution exists according to Theorem 2, find the coordinates of each corner point.*
- (2) *Construct a corner point table listing the value of the objective function at each corner point.*
- (3) *Determine the optimal solution(s) from the table in Step 2 (smallest=minimum, largest=maximum).*
- (4) *For an applied problem, interpret the optimal solution(s) in terms of the original problem.*

Example 1. Maximize and minimize $P = 30x + 10y$ subject to

$$2x + 2y \geq 4$$

$$6x + 4y \leq 36$$

$$2x + y \leq 10$$

$$x, y \geq 0$$

Example 2. Maximize and minimize $P = 3x + 5y$ subject to

$$x + 2y \leq 6$$

$$x + y \leq 4$$

$$2x + 3y \geq 12$$

$$x, y \geq 0$$

Applications.

Example 3. *An electronics firm manufactures two types of personal computers—a desktop model and a laptop model. The production of a desktop requires a capital expenditure of \$400 and 40 hours of labor. The production of a laptop requires a capital expenditure of \$250 and 30 hours of labor. The firm has \$20,000 capital and 2,160 labor-hours available for production of standard and portable computers.*

- (a) *What is the maximum number of computers the company is capable of producing?*
- (b) *If each desktop contributes a profit of \$320 and each laptop contributes a profit of \$220, how much profit will the company make by producing the maximum number of computers?*
- (c) *Does producing as many computers as possible produce the highest profit? If not, what is the highest profit and how many of each computer should be made in that case?*

Example 4. *A fruit grower can use two types of fertilizer in his orange grove, brand A and brand B. The amounts (in pounds) of nitrogen, phosphoric acid, and chloride in a bag of each brand are given in the table. Tests indicate that the grove needs at least 1,000 pounds of phosphoric acid and at most 400 pounds of chloride.*

	Brand A	Brand B
Nitrogen	8	3
Phosphoric Acid	4	4
Chloride	2	1

- (a) *If the grower wants to maximize the amount of nitrogen added to the grove, how many bags of each mix should be used? How much nitrogen will be added?*
- (b) *If the grower wants to minimize the amount of nitrogen added to the grove, how many bags of each mix should be used? How much nitrogen will be added?*